

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES
DETERMINISTIC MATHEMATICAL MODEL OF MATRIX'S STRESS-STRAIN
STATE OF WITH CONTINUOUS AND DISCONTINUOUS FIBERS**Korneev Andrey Mastislavovich^{1*}, Buzina Olga Petrovna², Sukhanov Andrey Vladimirovich³,
Abdullah Lutfi Saleh⁴ & Shipulin Ilya Andreevich⁵**
*1,2,3,4&5 Lipetsk state technical University, Russia**ABSTRACT**

The article provides an analysis of mathematical models according to diagnosis of the stress-strain state of anisotropic medium on the example of the construction, which is reinforced by continuous, discontinuous and discrete steel fibers. The authors, based on experimental data, developed a model, based on the terms of the normal section's balance. For the bent elements in the plane of the symmetry of transverse section, to calculate the strength for the deformation model a system of equations includes stiff characteristics, determined by the voltage in the matrix, which is reinforced by discrete fibers. It is suggested that the presence of general flaws in modern mathematical models that define the voltage values in the cross sections of the composites. A model based on the piecewise-linear characteristic curves is suggested, which allows to determine the voltage values in the composites, reinforced which are by discrete fibers of complex geometry.

Keywords: *Model of stress-strain state, Discrete fibers, Continuous fiber, Composite, The conditions of internal forces' equilibrium*

I. INTRODUCTION

Currently, researchers pay more attention to the problems of ensuring the reliability of buildings and structures. Structures' reliability primarily depend on the reliability of their structural elements, - jumpers, trusses, wall panels and columns. The choice of mathematical models is very important because they adequately describe the stress-strain state of such structures. The basis of such models make up the composite stress-strain diagram and fibers, which are an integral characteristic of the physical and mechanical features of materials and can be described by different mathematical expressions, which determines the accuracy of the calculated data. Computer technologies' use allows, with the help of nonlinear deformation model, to solve multifactorial problems, taking into account the actual structures' stress-strain state made of composites, reinforced by continuous and discrete fibers at all stages of work, and implementation of software methods of non-destructive check will make it possible to predict the structures' reliability on criteria: strength, deflection and cracking opening width.

II. COST OF DRIVING

As main mathematical models that describe the mechanics of destruction of bent structures of composites reinforced by continuous fibers (reinforcement), it is advisable to use a normative model according to modern standards and models' calculation based on real diagrams of material deformation.

Analysis of mathematical models on evaluation of the stress-strain state of anisotropic medium on the example of reinforced concrete structures has shown that nonlinear deformation model proposed by V.N. Baikov, N.I. Karpenko, B.S. Rastorguev [1, 2, 3] allows more accurately reflect the actual state of the composite elements under load. This model is based on terms of a normal section's equilibrium, broken into discrete areas of the composite matrix and continuous steel fibers. The account of physical nonlinearity of structures' work is done with the help of the mathematical description of diagrams of deformation of each of the composite and the use of stepper iterative method that implements the method of elastic solutions by A.A. Ilyushin. The method means that the solution of the nonlinear problem is obtained in the form of a sequence of linear problems, converging to the result. The equilibrium conditions of external and internal forces at any uploading are written as:

$$\begin{aligned} N_z &= \sum_n \sigma_{bn} A_{bn} + \sum_k \sigma_{sk} A_{sk} \\ M_x &= - \sum_n \sigma_{bn} A_{bn} x_n - \sum_k \sigma_{sk} A_{sk} x_k \\ M_y &= - \sum_n \sigma_{bn} A_{bn} y_n - \sum_k \sigma_{sk} A_{sk} y_k \end{aligned} \quad (1)$$

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where N_z – longitudinal force, M_x – bending moment in the X axis direction, M_y – bending moment in the Y axis direction.

Selected for the research method of describing of diagrams of materials' deformation is the most optimal, because it allows you to calculate the stresses in the matrix and steel fiber (reinforcement) by uniform dependencies on each stage of the short-term loading:

$$\begin{aligned} \sigma_b &= E_b \nu_b \varepsilon_b \\ \sigma_s &= E_s \nu_s \varepsilon_s \end{aligned} \quad (2)$$

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where σ_b , σ_s – respectively voltage in the matrix and fiber, E_b , E_s – materials' elastic modules, ε_b , ε_s – relative deformation.

According to the results of numerous experiments nowadays, both in our country and abroad, developed a large number of different ways of describing diagrams of straining matrix and a steel rod (continuous fibers) was developed, a lot of suggestions for their construction were put forward [4, 5]. In the works of V.V. Adischeva, V.N. Baykova, N.I. Karpenko and etc. [2] Analytical dependences for the description of material deformation diagrams $\sigma = f(\varepsilon)$ were suggested. Based on theoretical and experimental performed studies it was concluded that the approximation of actual diagrams of deformation with the help of spline functions or method of variable secant modulus is most satisfyingly used[2]:

$$\nu_b = \bar{\nu}_b \pm (\nu_0 - \bar{\nu}_b) \sqrt{1 - \omega_1 \eta - \omega_2 \eta^2} \quad (3)$$

where $\bar{\nu}_b$ – value of coefficient of change of matrix's secant modulus, ν_b ($0 < \nu_b < 1$) - on the top of the diagram, ($\bar{\nu}_b = \bar{\sigma}_b / (\varepsilon_{bR} E_b^0)$), $\bar{\sigma}_b = R_b$ – matrix's strength, ε_{bR} – relative deformation at maximum voltage, E_b^0 – the initial value of the elastic modulus of the matrix), η – the stress level, ($\eta = \sigma_b / \bar{\sigma}_b$, $0 < \eta < 1$); ν_0 – value of coefficient at the beginning of the diagram, ($\nu_0 = 1$ – building rising branch of the diagram and $\bar{\nu}_b = 2,05$ { $\bar{\nu}$ in the construction of the descending branch); ω_1 , ω_2 – parameters of curvature diagram defined by the formulas:

- for rising branch $\omega_1 = 2 - 2,5\bar{\nu}_b$, $\omega_2 = 1 - \omega_1$
- for descending branch $\omega_1 = 1,95 \{ \bar{\nu}_b - 0,138$, $\omega_2 = 1 - \omega_1$

Matrix's secant modulus of elasticity at any value of the voltage is determined by the formula:

$$E'_b = \frac{\sigma_b}{\varepsilon_b} = \nu_b E_b^0 \quad (5)$$

In some cases it is convenient to use dependences suggested in European regulations [4]:

$$\frac{\sigma_b}{R_b} = \frac{\left(\frac{E_b^0 \varepsilon_{bR}}{R_b} \right) \left(\frac{\varepsilon_b}{\varepsilon_{bR}} \right) - \left(\frac{\varepsilon_b}{\varepsilon_{bR}} \right)^2}{1 + \left(\frac{E_b^0 \varepsilon_{bR}}{R_b} - 2 \right) \frac{\varepsilon_b}{\varepsilon_{bR}}} \quad (6)$$

These analytical expressions used to describe with a high-reliability a diagram tension and compression of composites of low strength, in particular fine-grained matrix.

After making the appropriate changes the system (1) reduces to:

$$\begin{Bmatrix} N_z \\ M_x \\ M_y \\ \varepsilon_z \\ k_x \\ k_y \end{Bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \times \begin{Bmatrix} \varepsilon_z \\ k_x \\ k_y \end{Bmatrix}, \quad (7)$$

or

$$\{F\} = [R(\{F\}, S)] \times \{U\{F\}, S\}, \quad (8)$$

where $\{F\} = \{N_z, M_x, M_y\}^T$ – column vector of external forces, received, depending on the workload of the scheme $[R(\{F\}, S)]$ – stiff matrix for the normal cross-section, which is a function $\{F\}$ and S , whose elements are adjusted at each stage of loading; $\{U\{F\}, S\} = \{\varepsilon_z, k_x, k_y\}^T$ – column vector of strains obtained by solving the system of equations (8) [8].

Then, based on the hypothesis of a flat deformation, deformation calculated in the matrix and steel fibers for each discrete site:

$$\begin{aligned} \varepsilon_{bn} &= \varepsilon_z - k_x x_{bn} - k_y y_{bn} \\ \varepsilon_{sk} &= \varepsilon_z - k_x x_{sk} - k_y y_{sk} \end{aligned}, \quad (9)$$

where ε_z – deformation of the element at the longitudinal axis Z ; k_x, k_y – respectively values of curvature in X and Y axes; x_{bn}, y_{bn} – coordinates of the center of gravity of the discrete elements of the matrix; x_{sk}, y_{sk} – coordinates of the center of gravity of discrete elements of continuous steel fibers.

Deformation model for calculating the strength includes:

- The equation of equilibrium of external and internal forces in the normal section [2, 4]:

$$M_x = \sum_{i=y_{red}}^h \sigma_{bi} A_{bi} (V_i - y_{red}) + \sum_{i=1}^{y_{red}} \sigma_{bti} A_{bi} (y_{red} - V_i) + \sum_j \sigma_{sj} A_{sj} y_j, \quad (10)$$

$$N = \sum_{i=y_{red}}^h \sigma_{bi} A_{bi} + \sum_{i=1}^{y_{red}} \sigma_{bti} A_{bi} + \sum_j \sigma_{sj} A_{sj}; \quad (11)$$

- equations establishing strain distribution in the matrix and the continuous fiber on the normal section, based on the condition of the flat rotation and plane offset of section (hypotheses of flat sections):

$$\varepsilon_{bi} = \varepsilon_0 + \frac{1}{r_x} V_i, \quad \varepsilon_{sj} = \varepsilon_0 + \frac{1}{r_x} y_{sj}; \quad (12)$$

- equations that determine the relationship between stress and relative deformations of the matrix and continuous fibers in the composite:

$$\sigma_{bi} = f(\varepsilon_{bi}), \quad \sigma_{sj} = f(\varepsilon_{sj}). \quad (13)$$

In equations (10) - (13): M_x, N – the bending moment and the longitudinal force from an external load; A_{bi}, V_i, σ_{bi} and ε_{bi} – area, the distance from the bottom edges of the element stretched to the center of gravity of the i elementary layer composite matrix, stress and strain on the center of gravity level; $A_{sj}, y_{sj}, \sigma_{sj}$ and ε_s – area, the center of gravity coordinates of the j continuous fiber stress and strain in it; ε_0 – relative deformation of the fiber, which is located at the intersection of the selected axes X ; $1/r_x$ – curvature in the plane of action of the moment M_x .

For bent in the plane of symmetry of the cross section of the elements the equation system for calculating the strength on the deformation model is as follows [2, 8]:

$$\begin{aligned} M_x &= D_{11} \frac{1}{r_x} + D_{12} \varepsilon_0 \\ N &= D_{12} \frac{1}{r_x} + D_{22} \varepsilon_0 \end{aligned} \quad (14)$$

where D_{ij} ($i, j = 1, 2$) – stiff characteristics defined by the formulas

$$D_{11} = \sum_{i=y_{\text{red}}}^h A_{bi} (V_i - y_{\text{red}})^2 E_b v_{bi} + \sum_{i=1}^{y_{\text{red}}} A_{bi} (y_{\text{red}} - V_i)^2 E_{bt} v_{bt} + \sum_j A_{sj} y_j^2 E_{sj} v_{sj}, \quad (15)$$

$$D_{12} = \sum_{i=y_{\text{red}}}^h A_{bi} (V_i - y_{\text{red}}) E_b v_{bi} + \sum_{i=1}^{y_{\text{red}}} A_{bi} (y_{\text{red}} - V_i) E_{bt} v_{bt} + \sum_j A_{sj} y_j E_{sj} v_{sj}, \quad (16)$$

$$D_{22} = \sum_{i=y_{\text{red}}}^h A_{bi} E_b v_{bi} + \sum_{i=1}^{y_{\text{red}}} A_{bi} E_{bt} v_{bt} + \sum_j A_{sj} E_{sj} v_{sj}. \quad (17)$$

The coefficients of elasticity of i area of matrix v_{bi} , v_{bt} and continuous j fiber v_{sj} are determined by the formulas [2]:

$$v_{bi} = \frac{\sigma_{bi}}{E_b \varepsilon_{bi}}, v_{bt} = \frac{\sigma_{bti}}{E_{bt} \varepsilon_{bti}}, v_{sj} = \frac{\sigma_{sj}}{E_{sj} \varepsilon_{sj}}. \quad (18)$$

The voltage and the relative deformation of compressed and stretched matrix composite and steel continuous fiber σ_{bi} , ε_{bi} , σ_{bti} , ε_{bti} , σ_{sj} , ε_{sj} are determined from the above dependences for the phase diagrams of the matrix and steel fiber (3) - (6) [2].

To determine the stress and relative deformations of compressive and tension element of composite material, reinforced by discrete fibers, σ_{fbi} , ε_{fbi} , σ_{fbti} , ε_{fbti} it's necessary to conduct additional studies and relevant experiments to obtain mathematical dependences, $\sigma_{fbi}=f(\varepsilon_{fbi})$ и $\sigma_{fbti}=f(\varepsilon_{fbti})$, describing work of a composite material both in the elastic stage and in a crack stage where discontinuous fibers play the role of " bridges" preventing the formation of cracks in the normal section of a flexible element.

Reinforcing by discrete fibers load transfer is mainly in the boundary surfaces of the matrix and fibers. Important factors in this context are characteristic of the boundary surfaces of the matrix and the dispersed phase, diameter's dependence to length, elastic modules' dependence to the fiber matrix.

Location of discrete fibers in the matrix in real conditions is irregular. However, for convenience we can use the assumption that the reinforcement fiber is uniformly distributed throughout the matrix and oriented in the same direction. Discrete fiber is usually short and has the l length. On average, it can be assumed that all dragged-on in any section on equal conditions in terms of loading and failure probability. Through $\sigma_{f,m}$ we can denote the average value of tension σ_f of one discrete fiber in the longitudinal direction. If we use the average voltage caused by the load, then the σ_{fbt} voltage of the composite can be represented in cross section in the form of

$$\sigma_{fbt} = \sigma_{f,m} V_f + \bar{\sigma}_{bt} (1 - V_f), \sigma_{f,m} = \frac{1}{l} \int_0^l \sigma_f dx, \quad (19)$$

where V_f – volume content of the fibers in the composite.

If we input the maximum value in the dependence of $\sigma_{f,m}$ and use this dependence in the future, we obtain a formula for σ_{fbt} , where $\sigma_{f,max}$ is used. If we assume that is the voltage at which there is the destruction of the fiber, it is possible to determine the value R_{fbt} – composite tensile strength. Many researchers have studied the distribution of the fiber length. The following are the basic models used in these studies.

Cox [9] believed that thin discrete fiber of l length is enclosed in an elastic matrix and fiber connection with matrix is ideal. When creating stresses in the fiber, functioning in the axial direction, the strain at the interface of the matrix and fiber are identical. The ends of the fibers do not pass the stress. Under these conditions

$$\sigma_f = \frac{E_f - E_b}{E_b} \sigma_{fbt} \left\{ 1 - \frac{\operatorname{ch}\beta\left(\frac{l}{2} - x\right)}{\operatorname{ch}\left(\frac{\beta l}{2}\right)} \right\},$$

$$\sigma_{f \max} = (E_f - E_b) \left(1 - \operatorname{sch}\left(\frac{\beta l}{2}\right) \right) \varepsilon, \quad \varepsilon = \frac{\sigma_{fbt}}{E_b}, \quad (20)$$

$$\beta = \sqrt{\left\{ \frac{G_b}{E_f A_f} \frac{2\pi}{\ln\left(\frac{R}{r_0}\right)} \right\}},$$

where A_f – area of cross-section fibers, r_0 – radius of the fiber cross-section, $2R$ – the distance between the centers of the cross sections of the fibers, G_b – matrix shear strength, x – coordinate of the fiber's cross-section.

Another model was suggested by Autuoter [9]. He showed that the matrix surrounding discrete fiber shrinks when hardening, which leads to the appearance on the section's interface of matrix and compressive stress fibers. At the load's functioning in the direction of the fiber, friction forces appear on the interface, which determine the appearance of the σ_f fiber stress. If we use the μ coefficient of friction and the yield strength of the σ_{my} matrix, it is possible to determine σ_f :

$$\sigma_f = \mu \frac{\sigma_{my} t}{r_0^2} x, \quad (21)$$

where t – the thickness of the matrix layer.

The model suggested by Dow is a modification of the Cox model. In the place, where is fiber is located in Cox's model, a hole is made. Assuming that the original point is located in the center of the fiber having a $2l$ length, you can determine the σ_f value:

$$\sigma_f = \frac{P_b}{A_f + A_b \left(\frac{E_b}{E_f}\right)} \left[1 - \frac{\operatorname{ch}\left(\frac{\lambda x}{d_f}\right)}{\operatorname{ch}\left(\frac{\lambda l}{d_f}\right)} \right], \quad (22)$$

where λ – coefficient depending on the fiber diameter, modulus of elasticity and the content of the reinforcing material in the composite; A_b – area of matrix's cross-section; P_b – force applied to the matrix.

Rosen's model [9] is a modification of Dau model. In this model, the discrete fiber of $2l$ length and $2r_f$ diameter is surrounded by an adhesive material which is of $2l$ length and $2r_a$ radius. All of this is in a matrix of r_b radius. If we assume that the ratio of materials is ideal, then

$$\sigma_f = \frac{\sigma_{fbt} r_b^2 E_f}{E_b (r_b^2 - r_a^2) + E_f r_f^2} \left(1 - \frac{\operatorname{ch}(\eta x)}{\operatorname{ch}(\eta l)} \right),$$

$$\eta = \sqrt{\frac{2G_b}{E_a (r_a - r_f) r_f} \left(1 + \frac{E_f}{E_b} \frac{r_f^2}{r_b^2 - r_a^2} \right)}. \quad (23)$$

III. RESULTS & ANALYSIS

These models have common flaws: they describe the behavior of composites reinforced by discrete fibers that are oriented perpendicular to the normal plane of the section element. Moreover, these models are unapplicable in the description of the work of composites, reinforced by fibers of complex geometry that is different from a straight and cylindrical, - steel fiber with single and multiple limb at both ends, wavy fibers. The behavior of such composites is described in more complex equations.

The authors, based on experimental and theoretical issledovaniyahyu [6, 7], received based on piecewise linear characteristic curves, allowed to get more acceptable dependences defining σ_{fbt} values:

$$1) \quad \varepsilon_{fbt} = 0: \sigma_{fbt} = 0;$$

$$2) \quad 0 < \varepsilon_{fbt} < 0,24 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) \cdot 10^{-3}: \sigma_{fbt} = \frac{\varepsilon_{fbt}(E_b A_b + E_f A_f)}{A_b + A_f};$$

$$3) \quad \varepsilon_{fbt} = 0,24 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) \cdot 10^{-3}: \sigma_{fbt} = \frac{\varepsilon_{fbt}(E_b A_b + E_f A_f)}{A_b + A_f} \cdot \left(1 - \frac{7 \cdot 10^{-4}}{\mu_{fv}}\right);$$

$$4) \quad 0,24 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) \cdot 10^{-3} < \varepsilon_{fbt} \leq 2,4 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) \cdot 10^{-3}:$$

$$\sigma_{fbt} = \frac{0,24 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) \cdot 10^{-3} \cdot (E_b A_b + E_f A_f)}{A_b + A_f} + \varepsilon_{fbt} \left(\frac{\left(\sum_{j=1}^N P_{H,j}(k_A)^{\frac{3n}{r_{cp}}} \right) - 0,24 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) 10^{-3} (E_b A_b + E_f A_f)}{(A_f + A_b) (2,16 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) 10^{-3})} \right);$$

$$5) \quad 2,4 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) \cdot 10^{-3} < \varepsilon_{fbt} \leq 7,2 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) \cdot 10^{-3} \cdot W_H:$$

$$\sigma_{fbt} = \frac{\sum_{j=1}^N P_{H,j}(k_A)^{\frac{3n}{r_{cp}}}}{A_b + A_f} - \varepsilon_{fbt} \left(\frac{\sum_{j=1}^N 0,3 P_{H,j}(k_A)^{\frac{3n}{r_{cp}}}}{(A_f + A_b) (4,8 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) 10^{-3})} \right);$$

$$6) \quad 7,2 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) \cdot 10^{-3} \cdot W_H < \varepsilon_{fbt} \leq 14,4 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) \cdot 10^{-3} \cdot W_H:$$

$$\sigma_{fbt} = \frac{\sum_{j=1}^N 0,7 P_{H,j}(k_A)^{\frac{3n}{r_{cp}}}}{A_b + A_f} - \varepsilon_{fbt} \left(\frac{\sum_{j=1}^N 0,1 P_{H,j}(k_A)^{\frac{3n}{r_{cp}}}}{(A_f + A_b) (7,2 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) 10^{-3})} \right);$$

$$7) \quad 14,4 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) \cdot 10^{-3} \cdot W_H < \varepsilon_{fbt} \leq 19,2 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) \cdot 10^{-3} \cdot W_H:$$

$$\sigma_{fbt} = \frac{\sum_{j=1}^N 0,6 P_{H,j}(k_A)^{\frac{3n}{r_{cp}}}}{A_b + A_f} - \varepsilon_{fbt} \left(\frac{\sum_{j=1}^N 0,2 P_{H,j}(k_A)^{\frac{3n}{r_{cp}}}}{(A_f + A_b) (4,8 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) 10^{-3})} \right);$$

$$8) \quad 19,2 \cdot \sqrt[3]{R_{bt}(t)}(1 + 8V_f) \cdot 10^{-3} \cdot W_H < \varepsilon_{fbt}: \sigma_{fbt} = 0.$$

Here

$$W_H = 1 - 0,02 \{ \bar{\sigma}_b, P_H = (b)(0,55 + 0,015l_{f,an}) \}; \quad (24)$$

W_H, P_H – respectively the abscissa and ordinate of the first characteristic point of piecewise linear "load-displacement" diagram, which describes the work of the discrete fibers at the matrix's offset [7]; $l_{f,an}$ – sealing length of discrete fiber (mm), N – number of discrete fibers in the composite, n – number of discrete fibers located within a radius of 10 mm and affecting the operation of a single central fiber, r_{cp} – the average distance to the fibers located within 10 mm, k_A – coefficient which takes into account the work of the central fiber as a result of its influence on neighboring fibers and defined by the formula:

$$k_A = \left[(0,24 \cdot l_{f,an} - 2,7) \cdot \frac{\lambda_f^2}{\bar{\sigma}_b} + (7,7 - 0,52 \cdot l_{f,an}) \cdot \frac{\lambda_f}{\bar{\sigma}_b} + 1 \right], \lambda_f = l_{f,an,c} / l_{f,an}, \quad (25)$$

where $l_{f,an,c}$ – the average lengths' value of sealing of adjacent discrete fibers, mm

IV. CONCLUSION

To describe the stress-strain state of concrete applied modified depending offered N. I. Karpenko and C. Sujivorakul. Recent experimental and theoretical studies, based on piecewise linear characteristic curves, allowed to get acceptable dependences defining σ_{fb} values for models, that are applicable in the description of the work of composites, reinforced by fibers of complex geometry that is different from a straight and cylindrical, - steel fiber with single and multiple limb at both ends, wavy fibers

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